Stat 134: Section 11

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Conceptual Review

Consider a Poisson Process with rate λ per unit time. Identify what each random variable represents, and find the distributions of:

a. N_t;

b. W_k;

c. T_k. (How is this different from (b)?)

Problem 1

Suppose calls are arriving at a telephone exchange at an average rate of one per second, according to a Poisson arrival process. Find:

a. the probability that the fourth call after time $t = 0$ arrives within 2 seconds of the third call;

b. the probability that the fourth call arrives by time $t = 5$ seconds;

c. the expected time at which the fourth call arrives.

Ex 4.2.5 in Pitman's Probability

Problem 2: Gammas, Exponentials, and Moments

Consider the gamma function $\Gamma(r) = \int_0^\infty x^{r-1} e^{-x} dx, \;\; r > 0$.

a. Use integration by parts to show that $\Gamma(r+1) = r\Gamma(r)$.

b. Deduce from (a) that for any positive integer n, $\Gamma(n) = (n-1)!$

c. Show that if T Exp (1), then $\mathbb{E}(T^n) = n!$.

d. Show that if $S = T/\lambda$, then S Exp (λ). (Note: from this, we can easily show that $\mathbb{E}(S^n) = n!/\lambda^n$).

Hint: Consider the expression $P(S > s)$, then substitute for S appropriately.

Ex 4.2.9 in Pitman's Probability

Problem 3

Shocks occur to a system according to a Poisson process of rate λ. Suppose that the system survives each shock with probability α, independently of other shocks, so that its probability of surviving k shocks is α^k. What is the probability that the system is surviving at time t?

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