Stat 134: Section 14

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Conceptual Review

What functions have we used to characterize (i.e., fully describe) distributions of random variables? We have seen four.

Problem 1

Suppose R_1 and R_2 are two independent random variables with the same density function $f(x) = c\sin(x)$ when $x \in [0, \pi]$ and 0 in other case.

a. Find c such that f is a PDF Find the CDF of R_1.

b. Find the PDF and the CDF of $Y = \min(R_1, R_2)$.

Problem 2: Geometric from Exponential

Show that if $T \sim \text{Exp}(\lambda)$, then $Z = \text{int}(T) = \lfloor T \rfloor$, the greatest integer less than or equal to T, has a geometric (p) distribution on $\{0, 1, 2, \ldots\}$. Find p in terms of λ.
Ex 4.2.10 in Pitman's Probability

How can we use the CDF of Z to simplify this problem?

Problem 3

Let $U_{(1)}, \ldots, U_{(n)}$ be the values of n i.i.d. Uniform (0,1) variables arranged in increasing order. For $0 < x < y < 1$, find a simple formula for:

a. $P(U_{(1)} > x, U_{(n)} < y)$

b. $P(U_{(1)} > x, U_{(n)} > y)$

c. $P(U_{(1)} < x, U_{(n)} < y)$

d. $P(U_{(1)} < x, U_{(n)} > y)$

Ex 4.6.3 in Pitman's Probability