# *Stat 134: Section 16*

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## *Conceptual Review*

a. If $X, Y$ are independent with $N(\mu, \sigma^2)$ and $N(\lambda, \tau^2)$, then what is distribution of $aX + bY + c$?

b. What is chi-square distribution? Is that a distribution that we already know?

c. Let $X, Y$ have joint density $f_{X,Y}(x, y) > 0$ for all $x, y > 0$. Set up an integral that would yield the density of $Z = X + Y$ and $W = X/Y$.

d. Repeat (a), but for $Z = 3X + 4Y + 5$ and $W = 2X/Y + 3$.

## *Problem 1*

Let $X$ and $Y$ be independent standard normal variables. Find:

a. $P(3X + 2Y > 5)$

b. $P(\min(X < Y) < 1)$

c. $P(|\min(X < Y)| < 1)$

d. $P(\min(X, Y) > \max(X, Y) - 1)$

## *Problem 2*

Let $X \sim$ Unif (0,1), and $Y \sim$ Unif (0,2), independent of each other. Find the density of $Z = X + Y$, using:

a. the convolution formula;

b. the CDF of $Z$.

## *Problem 3: Competing Exponentials*

Suppose $X \sim$ Exp $(\lambda_X)$, $Y \sim$ Exp $(\lambda_Y)$, and $X, Y$ are independent.

a. Find $P(X < Y)$.

b. Find the density of $S = X + Y$, and using part (a), find the density of $T = X/Y$. (Hint: look at the CDF of $T$.)

c. Now suppose $\lambda_X = \lambda_Y = \lambda$, and find the distribution of $W = \frac{X}{X+Y}$. (Simplify $F_W$, and you should recognize $W$ as one of our famous distributions.)