Stat 134: Section 6

Adam Lucas

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Problem 1

Suppose the Stat department teaches 15 classes a semester: 2 have 60 students, 1 has 300 students, and 12 have 20 students. Each course is taught by a different professor, and each student only takes one class in the department.

a. For a randomly selected professor, what is the expected size of the class they teach?

b. For a randomly selected student, what is the expected size of the class they are in? How does this compare to part (a)?

Problem 2

Let A and B be independent events, with indicator random variables I_A and I_B.

a. Describe the distribution of $(I_A + I_B)^2$ in terms of $P(A)$ and $P(B)$.

b. What is $\mathbb{E}\left[(I_A + I_B)^2\right]$?

c. Suppose we now have a set of identical but not necessarily independent indicators $I_1, I_2, \ldots, I_n$. Derive a useful formula for $\mathbb{E}\left[(I_1 + I_2 + \ldots + I_n)^2\right]$.

Hint: Expand the polynomial, then use linearity of expectations.

Ex 3.2.10 in Pitman's Probability

Problem 3

Let I_A be the indicator of A. Show the following:

a. The indicator of A^c is $I_{A^c} = 1 - I_A$.

b. The indicator of the intersection AB of A and B is the product of I_A and I_B, $I_{AB} = I_A I_B$.

c. For any collection of events A_1, A_2,..., A_n, the indicator of their union is

$$I_{A_1 \cup A_2 \cup \cdots \cup A_n} = 1 - (1 - I_{A_1})(1 - I_{A_2}) \ldots (1 - I_{A_n}).$$

d. Expand the product in the last formula and use the rules of expectation to derive the inclusion-exclusion formula.

Ex 3.2.21 in Pitman's Probability

Prepared by Brian Thorsen and Yiming Shi