Stat 134: Section 16

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Conceptual Review

Let (X, Y) has joint density $f(x, y)$

a. How can we tell from the density function that X, Y are independent?

b. How do we find $P(A)$ for $A \subset \mathbb{R}^2$?

c. Now assume X, Y are independent standard Gaussian random variables. What can we say about $aX + bY + c$?

Problem 1

Let X and Y have joint density

$$f(x, y) = \begin{cases} 20(y-x)^3 & 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

a. Find $P(Y > 2X)$.

b. FInd the marginal density of X.

Problem 2

Let X and Y have joint density

$$f(x, y) = \frac{1}{2\pi} \exp\left[-\frac{1}{2}(x^2 + y^2)\right]\left\{1 + xy \exp\left[-\frac{1}{2}(x^2 + y^2 - 2)\right]\right\}.$$

a. Find marginal density of X and Y.

b. Conclude that even when marginal density of X,Y are Gaussian, (X, Y) may not be jointly Gaussian.